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scale free wireless networks**

DÁVID CSERCSIK

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Cooperation in traffic routing games on scale free wireless networks

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# Cooperation in traffic routing games on scale free wireless networks

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## Abstract

Local routing protocols in scale free networks have been extensively studied. In this paper we consider a wireless contextualization of this routing problem and analyze on the one hand how cooperation affects network efficiency, and on the other hand the stability of cooperation structures. Cooperation is interpreted as local exchange of topological information between cooperating agents, and the payoff of a certain node is defined based on its energy consumption during the routing process. We show that if the payoff of the nodes is the energy saving compared to the all-singleton case, basically coalitions are not stable. We introduce coalitional load balancing and net reward to enhance coalitional stability and thus the more efficient operation of the network. As in the proposed model cooperation strongly affects routing dynamics of the network, externalities will arise and the game is defined in a partition function form.

**Keywords:** partition function form games, networks, local routing

**JEL classification:** C71, L14, L96

# **Kooperatív routingjátékok skálafüggetlen vezeték nélküli hálózatokon**

Csercsik Dávid

## Összefoglaló

Skálafüggetlen hálózatok lokális routingprotokolljait széles körben tanulmányozták. Ebben a cikkben ennek a routingproblémának egy vezeték nélküli esetét tekintjük át, és egyfelől azt tanulmányozzuk, hogy a kooperáció hogyan befolyásolja a hálózat hatékonyságát, másfelől pedig a kooperáló struktúrák stabilitását vizsgáljuk. A kooperációt a lokális topológiai információ kölcsönös megosztásaként értelmezzük, és egy adott csomópont kifizetését energiateljesítmény felhasználása alapján definiáljuk. Megmutatjuk, hogy ha a csomópontok kifizetése a csupa singleton esethez képest energiamegtakarítás, akkor a koalíciók alapvetően nem stabilak. Bevezetjük a koalicionális terheléskiegyenlítést és a hálózati jutalmat a koalíciók stabilizálásának érdekében, mely biztosítja a hálózat hatékonyabb működését.

Mivel a javasolt modellben a kooperáció erősen befolyásolja a hálózati dinamikát, externáliák jelennek meg, így a játékot partíciós függvény formában definiáljuk.

**Tárgyszavak:** partíciós függvény típusú játékok, hálózatok, lokális routing

**JEL kód:** C71, L14, L96

# Cooperation in traffic routing games on scale free wireless networks

Dávid Csercsik<sup>\*†</sup>

June 22, 2014

## Abstract

Local routing protocols in scale free networks have been extensively studied. In this paper we consider a wireless contextualization of this routing problem and analyze on the one hand how cooperation affects network efficiency, and on the other hand the stability of cooperation structures. Cooperation is interpreted as local exchange of topological information between cooperating agents, and the payoff of a certain node is defined based on its energy consumption during the routing process. We show that if the payoff of the nodes is the energy saving compared to the all-singleton case, basically coalitions are not stable. We introduce coalitional load balancing and net reward to enhance coalitional stability and thus the more efficient operation of the network. As in the proposed model cooperation strongly affects routing dynamics of the network, externalities will arise and the game is defined in a partition function form.

**Keywords:** partition function form games, networks, local routing

**JEL-codes:** C71, L14, L96

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# 1 Introduction

Scale-free (SF) networks (Barabási and Albert, 1999; Barabási, Albert, and Jeong, 1999) are often used as a tool to describe the topology of communication networks (Albert, Jeong, and Barabási, 1999). In most cases it is a valid assumption that a certain node is not aware of the topology of the whole system. This may be especially true in wireless networks (Abolhasan, Wysocki, and Dutkiewicz, 2004; Hong, Xu, and Gerla, 2002; Garg, Aswal, and Dobhal, 2012; Mauve, Widmer, and Hartenstein, 2001) where the network topology itself is often subject to change because of fading due to changing environmental effects and potential mobility of the nodes. In such cases the delivery of packages may be performed by local routing protocols (Wang, Wang, Yin, Xie, and Zhou, 2006). Such routing problems in so called complex networks have been widely studied (for surveys see e.g Wang and Zhou (2006); Chen, Huang, Cattani, and Altieri (2011)). Although results corresponding to cooperative approaches in wireless networks can be found in the literature (see e.g. Khandani, Modiano, Abounadi, and Zheng (2005); Khandani, J.Abounadi, E.Modiano, and L.Zheng (2007); Ibrahim, Han, and Liu (2008)), these models usually do not consider SF network type fixed communication structure and are not focusing on local routing methods.

In the past years, game theoretic approaches in telecommunications (Douligeris and Mazumdar, 1992; Altman and Wynter, 2004; Altman, Boulognea, El-Azouzi, Jimenez, and L.Wynter, 2006) and wireless environment (Han, Niyato, Saad, Basar, and Hjorungnes, 2012; Al-Kanj, Saad, and Dawy, 2011) became more and more popular, including coalitional approaches as well (Saad, Han, Basar, Debbah, and Hjorungnes, 2009; Saad, Han, Debbah, Hjorungnes, and Basar, 2009a; Saad, Han, Debbah, and Hjorungnes, 2008; Saad, 2010; Saad, Han, Debbah, Hjorungnes, and Basar, 2009b; Pantisano, Bennis, Saad, Debbah, and Latva-aho, 2012).

Considering the more traditional game theory literature, networks (Jackson, 2008) and routing have been among the popular topics of the field recently, however usually selfish (Feldmann, Gairing, Lucking, Monien, and Rode, 2003; Roughgarden, 2005; Kontogiannis and Spirakis, 2005; Johari, Mannor, and Tsitsiklis, 2006) or competitive (Orda, Rom, and Shimkin, 1993; Cominetti, Correa, and Stier-Moses, 2006) routing models are considered, while coalitional approaches and models including externalities (Csercsik and Sziklai, 2012; Csercsik and Imre, 2013) are less representative.

In this article we propose a model to describe cooperation and analyze coalitional stability in wireless local routing SF network models. Basic traffic models considering local routing as (Wang, Wang, Yin, Xie, and Zhou, 2006) assume that the neighboring nodes are only aware of each others degree. Cooperation in the interpretation of the proposed model will mean that cooperating players exchange their local topological information (practically the list of their neighbors), which information will serve as a basis for packet routing. For the aim of simplicity, we assume that only neighboring nodes may cooperate, which implies that coalitions have to form complete graphs in the network.

In other words, while singleton models use first order routing while forwarding the packets (they look for the packet's destination only among their own neighbors), nodes in a coalition may search the neighbor list of some of their neighbors (their coalitional partners), and forward the packet according to this if match with the packet destination is found.

Since such exchange and utilization of second degree local information will affect the routing dynamics (e.g. in general it is straightforward to assume that packets will spend less time in the network if the routing efficiency is increased this way), cooperation will affect the energy consumption of nodes not taking part in the coalition. Since node payoffs will be defined based on individual energy consumption, this implies that externalities will appear, thus the game will be described in partition function form (Thrall and Lucas, 1963).

Partition function form games represent a novel approach for telecommunication problems, and they have been recently successfully applied for OFDMA (Orthogonal Frequency Division Multiple Access) problems in femtocell networks (Pantisano, Bennis, Saad, Verdone, and Latva-aho, 2011; Pantisano, Bennis, Saad, and Debbah, 2011; Pantisano, Bennis, Saad, Debbah, and Latva-aho, 2012).

## 2 Materials and methods

First of all, we assume that the nodes of the graph correspond to players or in other words agents, who may determine their strategy, namely they may choose to cooperate with other nodes or act selfishly. As mentioned, we will interpret our model in a wireless context where the transmission cost a single packet is proportional (in this case for the aim of simplicity equal) to the square of the distance. This will mean that we assign geometric positions to

the nodes of the graph, namely a coordinate pair in the unit square. Furthermore, during the generation of the communication graph we take spatial information into account as well.

For the generation of the network we use the geometry-modulated version of the Barabási-Albert algorithm (Barabási and Albert, 1999), as described in (Manna and Sen, 2002). A seed with  $n_{seed}$  nodes and  $m_{seed}$  link is used, and an iterative process is applied during which in each time step a new node with random position in the unit square is introduced and is randomly connected to  $m$  previous nodes. Any of these  $m$  links of the new node introduced at time  $t$  connects a previous node  $i$  with an attachment probability  $\pi_i(t)$  which is linearly proportional to the degree  $k_i(t)$  of the  $i$ -th node at time  $t$  and to  $l^\beta$ , where  $l$  denotes the Euclidean distance of the new node and node  $i$ , and  $\beta$  is a free parameter.  $\beta < 0$  corresponds to the case when nodes are more likely to connect closer ones. We call the resulting graph the *communication graph*.

$$\pi_i(t) \sim k_i(t)l^\beta \tag{1}$$

The basic traffic model based on (Wang, Wang, Yin, Xie, and Zhou, 2006) is described as follows: at each time step, there are  $R$  packets generated in the system with randomly chosen sources and destinations. We assume that each packet in the network holds information about its destination node. The buffer (queue) size of the nodes is assumed to be infinite, but any node can forward at most  $C$  (finite) packets in each time step. To make the model independent of the update order of the nodes, we assume that one packet can hop only once during a certain time step. To navigate packets, singleton nodes perform a local search. If the packet's destination is found among the neighbors, it is delivered directly to its target. Otherwise, it is forwarded to a chosen neighbor via the local routing mechanism.

In the current work we assume that the energy cost of cooperation (exchanging local topological information) can be neglected compared to the energy cost of packet forwarding. Nodes in a coalition perform first a local search, and if it is unsuccessful, they perform a second degree search among the neighbors of their coalitional partners as well. If a member of the actual coalition is found, which is adjacent to the packets destination, the packet will be forwarded to that node (since coalitions form complete graphs, this is always possible). In a coalition with three or more players it is possible that the packet destination is adjacent



to multiple coalitional members.<sup>1</sup> The next to nearest packet forwarding approach was already discussed by Tadić and Rodgers (2002), however not in a cooperative context.

If the destination is not found among the direct neighbors or among the neighbors of the coalitional partners of a node, the packet  $p$  is forwarded from node  $i$  to its neighbor  $j$  according to the preferential probability

$$\Pi_j = \frac{k_j^\alpha}{\sum_m k_m^\alpha} \quad (2)$$

where  $k_p$  denotes the degree of node  $p$ , the sum runs over the neighbors, and  $\alpha$  is a parameter describing the preference of high degree neighbours over low degree ones. As shown in (Wang, Wang, Yin, Xie, and Zhou, 2006),  $\alpha = -1$  is optimal regarding network congestion. Similar to (Wang, Wang, Yin, Xie, and Zhou, 2006), we assume that in a certain network none of the tokens may take the same edge again. There is a theoretical possibility that this assumption may lead to deadlock situations, but in practice the number of these scenarios is so low that they do not influence the results<sup>2</sup>.

We will monitor the overall network efficiency with the total energy consumption  $E_T$  (which is simply the sum of the energy consumption of individual nodes) and the average packet arrival time  $\bar{T}_{arr}$ . Naturally, as the results will show as well, these two indicators correlate, since if the packets reach their destination earlier, in general less transmission steps are required. Before the exact definition of the game, we introduce some examples to show how cooperation affects network dynamics.

## 3 Results

### 3.1 Example I

When considering network size for the demonstration of the results, on the one hand we have to take into account that we need a minimum level of complexity for the routing not to be trivial, and on the other hand we have to keep computations tractable and we have

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<sup>1</sup>A technical note: If we would perform the search according to the lexicographic ordering of the neighbors, the nodes with lesser index would be more loaded in such cases. To address this issue and equalize the load in such cases we always start the search from a random index among the coalitional neighbors.

<sup>2</sup>E.g. in a network of 300 nodes with R=25 during a simulation of 1000 steps, from the 25000 package only about an average of 40 become deadlocked.

to be able to visualize the results as well. The network of 30 nodes depicted in Fig. 1 was generated with parameters  $m = 3$ ,  $\beta = -2$  and a 10 node seed.

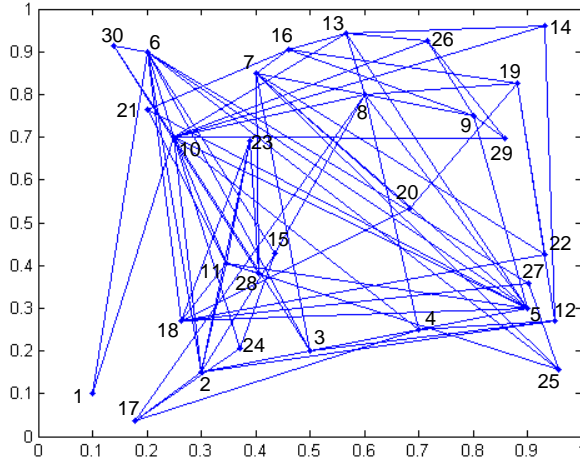


Figure 1: Network of example 1.

Regarding traffic dynamics, it is important to differentiate between congested and non congested cases. Following (Arenas, Díaz-Guilera, and Guimera, 2001) we define the congestion measure  $\eta$  as

$$\eta(R) = \lim_{t \rightarrow \infty} \frac{C}{R} \frac{\langle \Delta N_p \rangle}{\Delta t}$$

where  $\Delta N_p = N(t + \Delta t) - N(t)$  with  $\langle \dots \rangle$  indicates average over time windows of length  $\Delta t$  and  $N_p(t)$  represents the number of data packets present in the network at time  $t$ . If  $\eta(R)$  is significantly greater than zero (we can say that approximately  $\eta(R) > 0.25$ ), it indicates a congested state of the network (since the number of packets present in the network is steadily increasing). Although our aim in this article is not to determine the  $R_c$  values in various cases, we will use this indicator to describe non congested ( $R < R_c$ ,  $\eta \simeq 0$ ) and congested cases ( $R > R_c$ ,  $\eta > 0$ ).

If we simulate the traffic dynamics in a non-congested case with parameters  $\alpha = -1$ ,  $R = 5$   $C = 3$  Figure 2 depicts the energy usage results of nodes in 10 simulation.

On the one hand, it can be seen in Fig. 2 that the energy usage values are quite stable, the variance of the values is relatively low. This means that the average of several simulations can be regarded as a representative result. Furthermore it can be easily seen

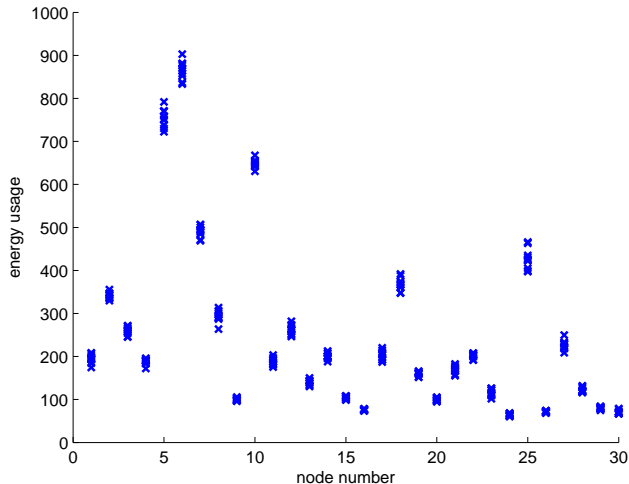


Figure 2: Energy usage of the nodes in all-singleton configuration. Simulation length: 1000 steps. Results of 10 simulations.

that, as expected, the energy consumption of the high degree nodes is high. The total energy used by the network is  $E_T = 7491.5$  in this case, while  $\bar{T}_{arr} = 4.54$ .

Next we analyze how coalition formation affects energy consumption values and network efficiency. Let us pick one coalition, e.g.  $\{5, 6, 18\}$ . First, it can be checked that it is a valid coalition, since nodes 5,6 and 18 form a  $G_3$  complete graph in the network. If we run the simulations according to the routing protocol defined in 2, we get the results depicted in Fig. 3.

Fig. 3 (see the averaged values in Table 1) shows that the energy consumption of the coalitional member nodes increased, while the energy consumption of all other nodes decreased. As we will see, this is not surprising. Let us consider an  $i$  member of the coalition  $C$ , who is forwarding a package with destination  $d$ . Let us furthermore suppose that  $d$  is in the neighborhood of  $j$ , which is an element of  $C$  as well. If no cooperation is present,  $i$  will forward this package randomly (taking into account only node degrees) to one of its neighbors ( $k$ ). This way the package will spend *at least* two more time steps in the network (if  $k$  is adjacent to  $d$  it may arrive in 2 steps), and in the future probably reach  $d$  via an undefined path, not necessary including  $j$ . In contrast, when the coalition  $C$  forms, the package will be forwarded to  $j$ . This means that coalitional members increase each others traffic load via applying the second order topological information. However as the

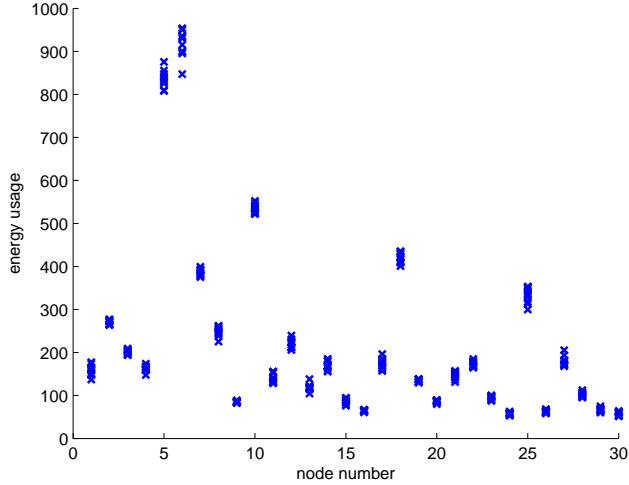


Figure 3: Energy usage of the nodes in the case when coalition  $\{5, 6, 18\}$  forms (all other). Simulation length: 1000 steps. Results of 10 simulations.

values  $E_T = 6651$  and  $\bar{T}_{arr} = 3.76$  show in this case, network efficiency is greatly increased even by the formation of this single coalition. In other words, the coalition formation resulted in a significant positive externality regarding all the remaining nodes.

Table 1 shows the energy consumption of individual nodes, the total energy usage of the system and average packet arrival time, in the case when various coalitions form.

We can see that, the dominant trend is validating our intuition - the energy consumption of the nodes which take part in coalitions significantly increases. In some exceptional cases (see e.g. the coalition  $\{2, 11, 23\}$ ), the the benefits implied by more efficient routing may overcome the handicap of increased coalitional load. In other words, this means that if we define the payoff of players and coalitions purely as the energy saving compared to the all singleton case, coalitions will not be stable in most of the cases. On the other hand, considering the  $E_T$  and  $\bar{T}_{arr}$  values, it can be clearly seen that coalition formation always enhances network performance, so from the point of view of network operation, it should be promoted.

According to these results we will introduce two modelling assumptions, and define the payoffs of nodes.

	$\emptyset$	{2, 3, 12}	{2, 6, 15}	{2, 11, 23}	{3, 5, 7}	{4, 8, 17}	{5, 6, 18}
1	195	179	169	174	176	189	160
2	341	364	404	333	285	328	269
3	260	236	235	243	258	260	207
4	188	174	171	178	168	189	164
5	755	699	676	691	783	742	826
6	866	797	914	782	726	845	898
7	491	453	433	450	579	479	378
8	294	282	259	274	264	330	255
9	100	96	91	93	85	96	83
10	649	594	584	610	580	641	541
11	188	172	165	185	159	179	139
12	263	303	242	242	224	250	222
13	140	128	128	129	119	134	117
14	201	188	184	188	179	193	174
15	105	98	116	97	90	101	91
16	76	71	71	71	67	74	64
17	203	193	185	195	179	225	175
18	368	344	337	345	318	362	414
19	160	149	147	149	139	155	132
20	100	95	93	97	89	98	83
21	169	154	153	160	152	163	147
22	201	185	193	195	180	200	169
23	116	106	105	116	106	114	93
24	64	64	57	64	59	66	58
25	427	394	387	385	374	414	332
26	71	71	68	67	65	70	61
27	227	211	213	214	196	215	171
28	122	116	112	108	110	122	99
29	80	74	73	76	71	78	69
30	71	69	66	66	61	71	58
$E_T$	7492	7060	7031	6977	6843	7382	6651
$\bar{T}_{arr}$	4.54	4.22	4.14	4.23	3.98	4.42	3.76

Table 1: Energy consumption of individual nodes, total energy usage of the system, and average packet arrival time in the case of various coalition formations.  $\emptyset$  means the all singleton coalition, in other cases only non-singleton coalitions are enumerated. Every result is an averaged value of 10 simulations.

### 3.2 Key assumptions and definition of the game

- First, we modify the routing protocol as follows. We introduce *coalitional load balancing* (CLB), which means that a parameter  $\sigma_1$  is defined to account for load relief of the coalitions. CLB works in the following way. If a member of a coalition is forwarding a packet, the destination of which can not be found neither among his own neighbors, nor the neighbors of among coalitional members, he will take into account the parameter sigma during the routing procedure. Namely the probability describing he will forward the package to his neighbor  $j$  will be

$$\Pi_j = \frac{k_j^\alpha}{\sum_m k_m^\alpha} \sigma \quad (3)$$

where  $\sigma = \sigma_1$  if  $j$  is in the coalition of  $i$  and  $\sigma = 1 - \sigma_1$  otherwise. This, in the case of  $\sigma_1 < 0.5$  will ensure that packets, who do not have their destination in the neighborhood of the coalition, will be probably turned away from it (in exchange for packets who have, will be drawn into it). We have to note that the inequality  $0 < \sigma_1$  shall be strict, because  $\sigma = 0$  may lead to deadlock and blocking situations as depicted in Fig. 4.

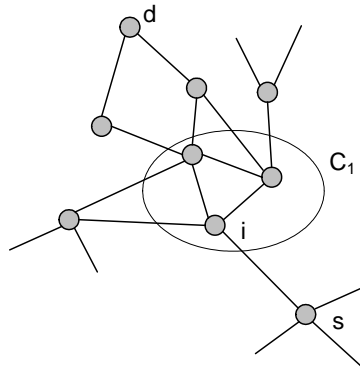


Figure 4: Coincidence of deadlock and blocking in the case of  $\sigma = 0$  and a packet with destination  $d$  arriving from  $s$  to  $i$ .

Consider a packet with destination  $d$  arriving from  $s$  to  $i$ . As  $i$  can not find  $d$  neither among his neighbors, nor among the neighbors of his coalition partners, he is theoretically forbidden to route it towards any coalitional partner. This leads to a deadlock situation. On the other hand, in the case of the given topology it can

be seen that node  $d$  is reachable from  $s$  only via coalition  $C_1$ , which means that the actual packet will never reach its destination, even if we suppose some more edges from  $i$  outward  $C_1$  and thus disregard the problem of deadlock.

- Second, we will assume an independent network operator, who is interested in efficient operation of the system. Furthermore we will assume that this network operator is able to reward, or in other words somehow compensate cooperating players for their increased traffic. If we suppose that nodes represent commercial mobile devices the most straightforward interpretation of this compensation is if we assume that this compensation can be included in the service fee. However, other means of compensation interpretation are also imaginable (e.g. the packets of cooperating nodes get priority in the routing process etc.). Formally we will assume that a good equivalent to the  $0 < p_{re} < 1$  part of the total energy saving of the system, compared to the all-singleton reference case (or its equivalent in some form), will be redistributed among the nodes, proportional to their actual traffic. This way the nodes who choose to cooperate and thus increase their traffic and enhance network performance, will gain more reward from the network operator. We will call this compensation *net reward*.

According to these considerations, during the routing protocol we apply CLB, and the payoff of single node  $i$  ( $v(i)$ ) is determined as his energy saving compared to the all singleton case plus the net reward. If according to these assumptions we repeat the simulations with  $\sigma_1 = 0.05$  for coalitions detailed in Table 1, and calculate nodal payoffs with  $p_{re} = 0.7$ , we get the results summarized in Table 2.

The first thing we can see in Table 2 is that all payoffs in the case of coalition formations are positive. This means that now (at least dominantly) superadditivity holds, which points toward the direction of coalitional stability. Second, the significant positive externalities still hold in all cases. Third, the application of CLB does not decrease network efficiency, in contrast the  $E_T$  and  $\bar{T}_{arr}$  values are slightly enhanced.

### 3.2.1 Coalitional stability

To get an impression about the stability of coalitions and about scenarios where multiple coalitions coexist let us analyze the stability of coalitions  $\{2, 6, 15\}$ ,  $\{3, 5, 7\}$  and  $\{4, 8, 17\}$ . To keep computations feasible, we restrict our calculations, and assume that potential deviators may not form coalitions with external nodes. Furthermore, for the same reason,

	$\emptyset$	{2, 3, 12}	{2, 6, 15}	{2, 11, 23}	{3, 5, 7}	{4, 8, 17}	{5, 6, 18}
1	0	30	45	33	43	8	59
2	0	46	30	56	93	13	101
3	0	66	57	40	62	7	83
4	0	23	32	28	40	59	51
5	0	117	143	123	112	38	101
6	0	138	112	146	225	31	145
7	0	73	97	77	37	27	160
8	0	36	52	44	63	63	73
9	0	13	17	15	26	6	28
10	0	103	127	98	149	35	186
11	0	29	44	28	50	13	66
12	0	21	48	34	69	18	67
13	0	21	22	16	33	13	37
14	0	24	33	27	49	12	51
15	0	15	22	15	22	5	30
16	0	9	11	9	17	4	21
17	0	25	38	29	41	97	51
18	0	45	70	50	82	12	42
19	0	17	26	23	38	11	43
20	0	12	15	11	19	5	25
21	0	19	31	24	32	7	43
22	0	27	32	28	37	8	53
23	0	15	23	31	23	3	33
24	0	7	10	8	12	2	14
25	0	67	91	73	102	24	141
26	0	10	11	10	17	2	20
27	0	29	40	29	47	7	66
28	0	15	24	15	24	9	35
29	0	10	17	11	18	3	21
30	0	9	15	9	17	3	22
30	71	69	66	66	61	71	58
$E_T$	7492	6957	6823	6921	6692	7219	6556
$\bar{T}_{arr}$	4.54	4.14	4	4.14	3.87	4.38	3.73

Table 2: Nodal payoffs, total energy usage of the system, and average packet arrival time in the case of various coalition formations.  $\emptyset$  means the all singleton coalition, in other cases only non-singleton coalitions are enumerated. Every result is an averaged value of 10 simulations.



Coalitions	v(2)	v(3)	v(4)	v(5)	v(6)	v(7)	v(8)	v(15)	v(17)	$E_T$	$\bar{T}_{arr}$
{2, 6, 15}, {3, 5, 7}, {4, 8, 17}	105	104	97	231	291	101	143	39	129	5931	3.32
{2, 6}, {3, 5, 7}, {4, 8, 17}	109	104	100	234	307	108	140	39	131	5883	3.29
{2, 15}, {3, 5, 7}, {4, 8, 17}	112	71	86	141	255	61	119	26	120	6278	3.63
{6, 15}, {3, 5, 7}, {4, 8, 17}	116	81	89	165	252	79	125	24	125	6202	3.54
{3, 5, 7}, {4, 8, 17}	98	64	87	131	231	39	105	26	113	6401	3.71
{2, 6, 15}, {3, 5}, {4, 8, 17}	53	80	83	184	181	130	118	30	124	6335	3.68
{2, 6, 15}, {3, 7}, {4, 8, 17}	72	58	92	244	217	113	125	31	124	6186	3.56
{2, 6, 15}, {5, 7}, {4, 8, 17}	75	95	92	171	237	85	131	36	126	6129	3.45
{2, 6, 15}, {4, 8, 17}	32	60	80	166	124	104	111	25	119	6524	3.82
{2, 6, 15}, {3, 5, 7}, {4, 8}	105	106	78	214	288	97	120	37	70	6040	3.37
{2, 6, 15}, {3, 5, 7}, {4, 17}	100	103	82	203	281	94	106	36	85	6059	3.38
{2, 6, 15}, {3, 5, 7}, {8, 17}	108	99	71	218	296	99	125	39	100	5985	6.03
{2, 6, 15}, {3, 5, 7}	97	101	66	212	275	94	123	39	100	6033	3.38

Table 3: Nodal payoffs in various coalitional structures. The values are averaged results of 10 simulations.

Partition	Coalitional values
{2,6,15}	435
{2,6}, {15}	416 , 39
{2,15}, {6}	138 , 255
{2},{6,15}	116 , 276
{2},{6},{15}	98,231,26

Table 4: Partition function of coalition {2, 6, 15}

we assume that only one coalition may break up in the same time. According to this, the node relevant payoffs, from which the the values of the partition functions can be calculated by summing over the coalitions, are summarized in Table 3.

Let us consider coalition {2, 6, 15} first. In this case, if we restrict ourselves to this residual game, the partition function will be as summarized in Table 4

If we assume transferable utility (which may be realistic assumption e.g. in the case of mobile commercial devices where the players may be compensated for higher energy consumption via lower service fees), and calculate the (pessimistic) recursive core (Kóczy, 2007) for the partition function presented in Table 4, we find that the partition {2, 6}, {15}

is stable, with the payoff configuration

$$x(2) + x(6) = 416 \quad x(15) = 39 \quad 116 < x(2) < 161 \quad (4)$$

If we take a closer look on the  $E_T$  and  $\bar{T}_{arr}$  values in Table 3, we can see that (assuming that the members of the other coalitions do not deviate) this partition of  $\{2, 6, 15\}$  results in the most efficient operation of the network.

Regarding  $\{3, 5, 7\}$  and  $\{4, 8, 17\}$  we find that in both cases the grand coalitions are stable as depicted in Fig. 5. Again we can see in Table 3 that the partitions in which  $\{3, 5, 7\}$  and  $\{4, 8, 17\}$  form the grand coalition are the most efficient.

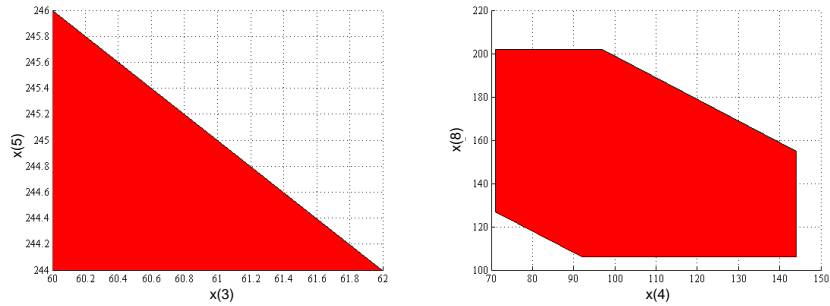


Figure 5: Recursive cores of coalitions  $\{3, 5, 7\}$  and  $\{4, 8, 17\}$  in the payoff space. In the first case the equality  $x(3) + x(5) + x(7) = 436$ , in the second the equality  $x(4) + x(8) + x(17) = 369$  holds

Appendix A holds further data underlining the trend that stable coalitional configurations correspond to the most efficient operation modes of the network.

### 3.3 Example II

In this example we use a network with 300 nodes to analyze how the efficiency enhancing effect of coalition formation scales up. Due to the limitations of computing capacity in this case we do not analyze coalitional stability, only how the various coalitional configurations affect the performance indicators. Based on the previous results, we assume that coalition structures which imply the most high performance network operational modes are dominantly stable, if the net reward is high enough.

The network used in this example was generated with a seed of 10 nodes,  $m = 4$  and  $\beta = -2$ .

### 3.3.1 Non-congested case

First we analyze the network performance without congestion. We use the parameters  $C = 7$  and  $\alpha = -1$ ,  $R = 10$ . Results are summarized in Table 5.

Coalition structure	$E_T$	$\bar{T}_{arr}$	$\eta$
$\emptyset$	51126	34.81	0.07
5x2	48579	33.28	-0.01
10x2	45307	30.90	-0.03
20x2	41078	28.05	-0.04
50x2	35078	23.88	0.03
100x2	31564	21.57	-0.02
1x5 2x4 7x3	38963	26.38	0.05
1x5 2x4 17x3	34705	23.07	0.01
1x5 2x4 27x3	33445	22.37	-0.08
1x5 2x4 31x3 16x2	31492	20.85	0
1x5 2x4 31x3 66x2	29209	19.13	0.02

Table 5: Network performance at various levels of cooperation and various coalitional structures. The column 'Coalition structures' indicates the number of different size coalitions (e.g. '1x5 2x4 31x3 66x2' indicates 1 coalition of size 5, 2 of size 4 etc.).

Table 5 shows that (as expected) as the level of cooperation increases, the network performance is enhanced. Simulation results show that this performance increase can be very significant. The presence of larger size coalitions implies further growth in network efficiency. The  $\eta$  values which are practically equal to 0 show that no congestion appears.

### 3.3.2 Congested case

Since the measure of congestion  $\eta$  is also a function of coalitional structure

Regarding the performance indicators, the results are similar to the non-congested case, except that the benefits of cooperation in the congested case are even more prominent. On

Coalition structure	$E_T$	$\bar{T}_{arr}$	$\eta$
$\emptyset$	90421	103.29	1.86
5x2	89214	98.79	1.83
10x2	88279	91.79	1.54
20x2	84368	74.85	1.12
50x2	77642	45.26	0.5
100x2	73470	35.37	0.19
1x5 2x4 7x3	79237	71.52	1.07
1x5 2x4 17x3	75886	55.61	0.74
1x5 2x4 27x3	73792	47.62	0.52
1x5 2x4 31x3 16x2	71771	35.1	0.35
1x5 2x4 31x3 66x2	67934	29.65	0.27

Table 6: Network performance at various levels of cooperation and various coalitional structures. The column 'Coalition structures' indicates the number of different size coalitions (e.g. '1x5 2x4 31x3 66x2' indicates 1 coalition of size 5, 2 of size 4 etc.).

the other hand, if we analyze the  $\eta$  values in Table 6 we can see that increasing level of nodal cooperation alleviates network congestion as well. Very high levels of cooperation almost eliminate network congestion.

## 4 Conclusions and future work

We introduced a game theoretic model to describe coalitional formation in wireless networks with fixed communication structure and analyzed the implied phenomena on scale-free topology. Cooperation was interpreted as exchange of local topological information. We have shown that if we define the payoffs of the nodes exclusively by the energy saving compared to the non-cooperative case, players are not motivated to form coalitions, since the traffic of such cooperating agents increase. To enhance coalitional stability and retain positive externalities we introduced coalitional load balancing and net reward, and calculated the payoff of nodes according to these assumptions. This means that these modifications allow us to motivate the players for cooperation and to enhance network performance in the same time. Furthermore we have shown that increasing levels of coop-

eration ease network congestion.

There are several directions in which the current concept may be extended. First, as we consider a wireless environment, it is straightforward to assume that the nodes (or at least some of the nodes) are moving. In this case the stability of coalitions may be subject to change due to change in the transmission costs. Second, in the current model we assumed that the exchange of local topological information is free, or it can be neglected compared to the energy cost of packet forwarding. To make the model more realistic one may assume that the exchange of local topological information itself takes place via packet forwarding, thus its energy cost may be incorporated in the model. Third, the concept of net reward may be refined as well. E.g. it can be assumed that the network operator holds a few high degree nodes (e.g. with fixed position - base stations), and is able to redistribute only the energy savings corresponding to this nodes among the players.

## 5 Acknowledgements

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## Appendix A

To give some further impression into coalitional stability of the model we analyze some more cases. Let us consider the coalitions  $\{5, 7, 9\}$ ,  $\{10, 13, 14\}$  and  $\{11, 18, 23\}$  and the values summarized in Table 7.

Considering  $\{5, 7, 9\}$ , the stability analysis shows that  $\{5, 7\}$ ,  $\{9\}$  is the stable partition with  $x(5) + x(7) = 373$ ,  $x(9) = 38$  and  $233 < x(5) < 235$ . Considering  $\{10, 13, 14\}$  and  $\{11, 18, 23\}$  the grand coalitions are stable, with payoffs depicted in Fig. 6. Again, it can be seen that stable partitions correspond to the most efficient network operation modes.

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Coalitions	v(5)	v(7)	v(9)	v(10)	v(11)	v(13)	v(14)	v(18)	v(23)	$E_T$	$\bar{T}_{arr}$
{5, 7, 9}, {10, 13, 14}, {11, 18, 23}	236	116	36	215	81	56	57	117	47	5999	3.35
{5, 7}, {10, 13, 14}, {11, 18, 23}	244	129	38	217	79	58	59	123	47	5961	3.33
{5, 9}, {10, 13, 14}, {11, 18, 23}	229	138	22	164	62	42	40	79	41	6305	3.66
{7, 9}, {10, 13, 14}, {11, 18, 23}	233	137	30	159	63	44	41	82	40	6274	3.63
{10, 13, 14}, {11, 18, 23}	181	101	24	110	50	35	29	57	33	6593	3.85
{5, 7, 9}, {10, 13}, {11, 18, 23}	224	113	36	206	78	45	73	116	47	6046	3.41
{5, 7, 9}, {10, 14}, {11, 18, 23}	195	102	32	182	72	42	32	102	43	6203	3.51
{5, 7, 9}, {13, 14}, {11, 18, 23}	180	86	30	191	71	47	40	101	43	6266	3.54
{5, 7, 9}, {11, 18, 23}	193	94	31	196	68	49	41	102	42	6232	3.51
{5, 7, 9}, {10, 13, 14}, {11, 18}	222	108	36	204	73	54	55	116	41	6052	3.4
{5, 7, 9}, {10, 13, 14}, {11, 23}	179	84	34	183	66	48	45	113	35	6229	3.51
{5, 7, 9}, {10, 13, 14}, {18, 23}	185	86	34	191	64	52	48	111	36	6207	3.49
{5, 7, 9}, {10, 13, 14},	184	90	35	178	65	50	48	111	34	6221	3.51

Table 7: Nodal payoffs in various coalitional structures. The values are averaged results of 10 simulations.

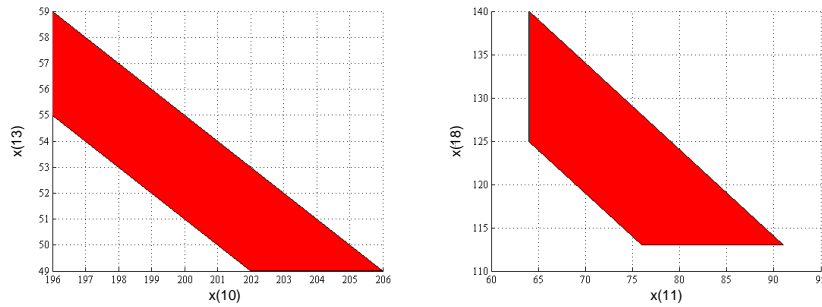


Figure 6: Recursive cores of coalitions  $\{10, 13, 14\}$  and  $\{11, 18, 23\}$  in the payoff space. In the first case the equality  $x(10) + x(13) + x(14) = 328$ , in the second the equality  $x(10) + x(13) + x(14) = 245$  holds.

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